DOCUMENT RESUME

ED 139 663 SE 022 556

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TITLE Math Diagnosis and Prescription Literature Search: Final Report.
INSTITUTION Research for Better Schools, Inc., Philadelphia, Pa.
SPONS AGENCY National Inst. of Education (DHEW), Washington, D.C.
PUB DATE 15 Dec 76
NOTE 42p.; For related document, see SE 022 555; Contains light and broken type

EDRS PRICE MF-$0.83 HC-$2.06 Plus Postage.
DESCRIPTORS *Annotated Bibliographies; Basic Skills; Bibliographies; *Educational Diagnosis; *Elementary School Mathematics; Elementary Secondary Education; Error Patterns; Instruction; *Literature Reviews; Mathematics Education; Research Reviews (Publications)

ABSTRACT

The procedures used for locating documents on diagnosis and prescription in mathematics are described, and the coding system used in reporting the documents is explained. Appendices include lists of basic documents, basic sources, frequently cited authors, and relevant phrases and concepts; a partial bibliography of documents concerned with diagnosis and prescription in mathematics; an outline of document codes; and an annotated bibliography of selected documents. (DT)

MATH DIAGNOSIS AND PRESCRIPTION LITERATURE SEARCH: FINAL REPORT

Submitted to

National Institute of Education

by

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and

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December 15, 1976

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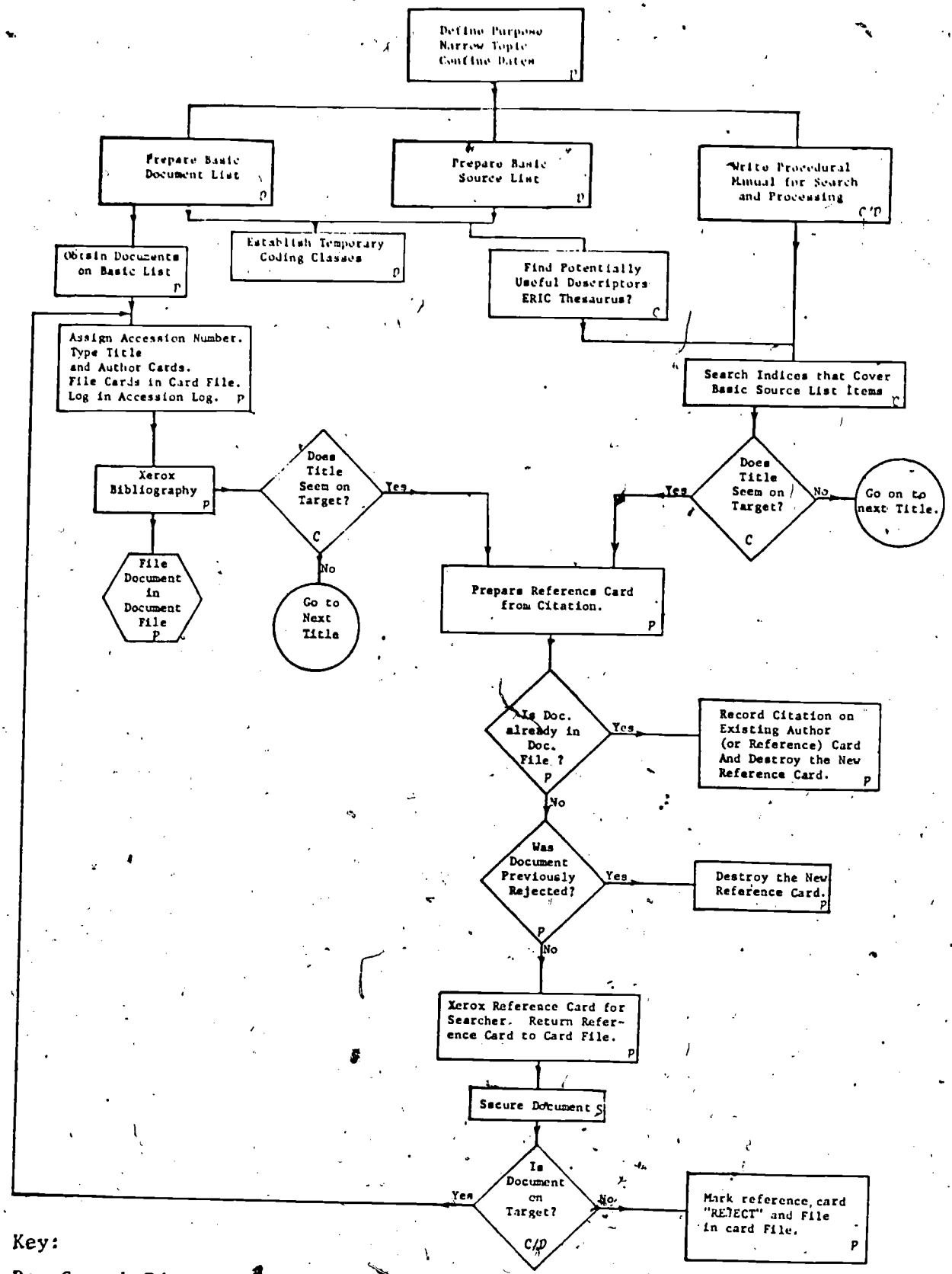
MATH DIAGNOSIS AND PRESCRIPTION
LITERATURE SEARCH:
FINAL REPORT

One aspect of mathematics teaching that theorists, researchers, and practitioners have identified as important is that of diagnosis and prescription. The workscope on this topic focused on mathematics diagnostic-prescriptive classroom activities of K-8 teachers. Plans outlined in the original proposal to NIE called for knowledge to be collected from three sources: (1) a literature search, (2) a classroom survey, and (3) a small scale study of systematic errors. The original plan for the work unit also included an activity that would have explored methods for utilizing the recommendations of local teachers, teacher trainers, mathematicians, and mathematics coordinators for the improvement of teacher diagnostic-prescriptive skills. The literature search was designed to provide a knowledge-base for any mutually agreed upon intervention or research activities.

The literature search was initiated in September 1976, but energies were diverted in mid-November when word was received from NIE indicating its interest in a specific classroom survey. The procedures used in the literature search and its status as of November 30, 1976 are reported in the remainder of this document.

The literature search utilized the traditional strategy of examining selected indices, bibliographies, and yearbooks as well as the process of citation searching. Figure 1 is a flow chart of the procedures developed and followed to date.



Key:

D: Search Director
C: Search Coordinator
S: Searcher
P: Processor

Figure 1

One of the early tasks was the identification of key documents, referred to on the diagram as Basic Documents, from which citation searching would begin (See Appendix A for the list of Basic Documents). Bibliographies of these documents were examined, the works of citations that appeared to be relevant provided further citations, and so on until an exhaustive list of pertinent books and articles could be obtained. The title of every citation was evaluated; those considered promising were recorded, secured and read. If the item was then determined to be useful to the study, an accession number was assigned, an author card prepared, and the document filed. In addition, its bibliography was copied, and the steps of citation analysis were thus initiated once again.

Some literature was traced through the indices and bibliographies referred to on the flow chart as Basic Sources. (Listed in Appendix A.) The bibliographies of pertinent articles located through these sources were then included in the citation searching procedure.

In this effort, the search coordinator was assisted by a list of authors whose writings frequently pertain to diagnosis and prescription in K-8 mathematics, and a list of phrases and concepts which are likely to appear in titles of relevant articles. (See Appendix A.)

Many recent articles on "readiness" and use of interviews focus on the Piagetian stages of development. Those which merely describe interviews for assessing the four stages were not included unless the implications for teaching were reported. Articles were deemed pertinent if they were reporting <u>research</u> indicating a particular stage to be a prerequisite for learning a specific math skill or concept. The intent was to avoid collecting all Piagetian studies that were to replicate Piaget's findings

or intended to demonstrate whether or not concepts such as conservation of number can be taught. Relevance to diagnosis and prescription in the classroom was the primary criterion.

Some articles deal with diagnosis and/or prescriptions related to special education. If the described diagnosis or diagnostic procedure is for the purpose of determining the nature or extent of the handicap, or if the prescription rationale is relevant only to the handicap (use of Braille writer, etc.), then the article was rejected. Articles dealing with remedial clinical settings were also included if they described a strategy, technique, instrument, or data that might be used, or adapted for use, in the classroom setting.

A dual system of coding was established to permit identification of each document by type of publication (e.g., research report, instructional materials, and position paper) and by content (e.g., factors contributing to learning difficulties, and models of classroom diagnostic-prescriptive teaching). The codes are listed as two or more separate decimals. Decimals are used to allow for sub-classification of both type and content codes. For each document, codes in the A row refer to type of publication; B row codes signify content. A legend of A and B codes is presented in Appendix C.

The coding system works as follows:

McAloon, Ann. "Some Issues of Concern in the Development of a Diagnostic Testing Program." From Proceedings of the Third National Conference on Remedial Mathematics, pp. 92-98. Kent, Ohio: Kent State University, 1976.
Code: A. 3.0, 5.0
B. 3.3, 8.0

The Code A legend (Appendix C) indicates that this document is both a description of instructional procedure and a position paper. The Code B

legend (Appendix C) shows that the document deals with the use of inventories for diagnosis and discusses teacher education in diagnostic-prescriptive teaching.

By November 30, fifty-nine documents pertinent to the study had been secured and filed. A list of these documents appears in Appendix B; annotations for forty-seven of them are in Appendix D. An additional 109 documents have been identified for further evaluation. While the present collection is in a sense incomplete, it nevertheless encompasses much of the important literature pertaining to the diagnosis and prescription of mathematics for grades K-8. It therefore has potential as a survey of the most important research and theories to date, and as a guide to further resources. It is RBS' intention to use some of the material in a larger literature search. In addition, RBS hopes that this study will be of value to others who share its concern for providing children with sound mathematics instruction.

APPENDIX A

BASIC DOCUMENTS

BASIC SOURCES

FREQUENTLY CITED AUTHORS

RELEVANT PHRASES AND CONCEPTS

Basic Documents
Basic Sources
Frequently Cited Authors
Relevant Phrases and Concepts

Basic Documents

Ashlock, Robert B. Error Patterns in Computation: A semi-Programmed Approach. Columbus, Ohio: Charles E. Merrill Publishing Co., 1976.

Brueckner, Leo J. "Diagnosis in Arithmetic." In Educational Diagnosis, the Thirty-fourth Yearbook of the National Society for the Study of Education, pp. 269-302. Bloomington, Ill.: Public School Publishing Company, 1935.

Cox, Linda S. Analysis, Classification, and Frequency of Systematic Error Computational Patterns in the Addition, Subtraction, Multiplication, and Division Vertical Algorithms for Grades 2-6 and Special Education Classes. Final report, U.S. Department of Health, Education & Welfare, National Institute of Education Contract #SE018001. Kansas City: University of Kansas Medical Center, June 1974.

Dutton, Wilbur H. Evaluating Pupils' Understanding of Arithmetic. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1964.

Glennon, Vincent J., and John W. Wilson. "Diagnostic-Prescriptive Teaching." In The Slow Learner in Mathematics, the Thirty-fifth Yearbook of the National Council of Teachers of Mathematics, edited by W. C. Lowry, et al., pp. 282-318. Washington, D.C.: National Council of Teachers of Mathematics, 1972.

Lankford, Francis G., Jr. Some Computational Strategies of Seventh Grade Pupils. Final Report, U.S. Department of Health, Education, and Welfare, Office of Education National Center for Educational Research and Development (Regional Research Program) and The Center for Advanced Study, The University of Virginia, Project Number 2-C-013, Grant Number OEG-3-72-0035. Charlottesville, Virginia: University of Virginia, October, 1972.

National Conference on Remedial Mathematics. (Selected papers from the first and third conferences).

Peck, Donald M., and Stanley M. Jencks. "What the Tests Don't Tell," The Arithmetic Teacher, XXI (January, 1974), pp. 54-56.

Reisman, Fredericka K. A Guide to the Diagnostic Teaching of Arithmetic. Columbus, Ohio: Charles E. Merrill Publishing Company, 1972.

Roberts, Gerhard H. "The Failure Strategies of Third Grade Arithmetic Pupils," The Arithmetic Teacher, XV (May, 1968), pp. 442-446.

Trueblood, Cecil R. "A Model for Using Diagnosis in Individualizing Mathematics Instruction in the Elementary School Classroom," The Arithmetic Teacher, XVIII (November, 1971), pp. 505-511.

Tyler, Ralph W. "Elements of Diagnosis." In Educational Diagnosis, the Thirty-fourth Yearbook of the National Society for the Study of Education, pp. 113-129. Bloomington, Ill.: Public School Publishing Company, 1935.

Weaver, Fred J. "Big Dividends from Little Interviews," The Arithmetic Teacher, II (April, 1955), pp. 40-47.

Basic Sources

Journal of Research in Mathematics Education

The Arithmetic Teacher

"An Evaluation of Journal-Published Research Reports on Elementary School Mathematics, 1900-1965," Marilyn N. Suydam. Doctoral dissertation, The Pennsylvania State University, 1967.

"Annotated Compilation of Research on Secondary School Mathematics, 1930-1970, Volume 1," Marilyn N. Suydam. Office of Education, U.S. Department of Health, Education and Welfare, Regional Research Program. The Pennsylvania State University, 1972.

"Interpretive Study of Research and Development in Elementary School Mathematics, Vol. 2," Marilyn N. Suydam. Office of Education, U.S. Department of Health, Education and Welfare, Bureau of Research. The Pennsylvania State University, 1969.

Frequently Cited Authors

Ashlock, Robert

Brownell, William A.

Brueckner, Leo J.

Bunwell, Guy T.

Callahan, Leroy

Cox, Linda S.

Dupton, Wilbur

Grossnickle, Foster E.

Heddens, James

Lankford, Francis G.

Morton, Robert L.

Underhill, Richard

Uprichard, Edward

Weaver, Fred

Wilson, John

Relevant Phrases and Concepts

Clinical (Intervention) Research

Clinical Studies

Common Errors

Computational Strategies

Continuous Progress Model (where diagnosis is treated)

Diagnosis/Diagnostic

Instruments

Interviews

Case Studies

Difficulties in _____ (math topic)

Evaluation

Diagnostic

Reasons for

Uses of

Failure Strategies

Group Diagnosis

Grouping/Grouping Practices (basis for)

Interview Technique

Kinds of Errors

Mastery Learning/Mastery Model (where diagnosis is treated)

Patterned Errors

Pupils understanding of _____ (math topic)

Readiness (if specific prerequisite skills are discussed)

Remedial Work/Remediation

Planning for

Rationales

Systematic Errors

Testing (diagnostic)

APPENDIX B

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* Ashlock, Robert B. "Underachievers in Elementary Mathematics: Diagnosing and Correcting Difficulties in Computation." Paper presented at the 51st Annual Meeting of the National Council of Teachers of Mathematics, Houston, Texas, April, 1973.

Ashlock, Robert B., John W. Wilson, and Barton Hutchings. "Identifying and Describing the Remedial Mathematics Student." Paper presented at The First National Conference on Remedial Mathematics, Kent State University, Kent, Ohio, 1974.

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* Annotation may be found in Appendix D.

* Brueckner, Leo J. "The Principles of Developmental and Remedial Instruction." In Educational Diagnosis, the Thirty-fourth Yearbook of the National Society for the Study of Education, pp. 189-198. Bloomington, Ill.: Public School Publishing Company, 1935.

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* Carpenter, Thomas P., et al. "Notes from National Assessment: Perimeter and Area," The Arithmetic Teacher, XXII (November, 1975), pp. 586-590.

* Carpenter, Thomas P., et al. "Subtraction: What Do Students Know?" The Arithmetic Teacher, XXII (December, 1975), pp. 653-657.

* Collins, Kenneth M. "An Investigation of the Variables of Bloom's Mastery Learning Model for Teaching Mathematics." Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, Ill., April, 1972.

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* Cox, Linda S. "Diagnosing and Remediating Systematic Errors in Addition and Subtraction Computations," The Arithmetic Teacher, XXII (February, 1975), pp. 151-157.

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* Annotation may be found in Appendix D.

APPENDIX C

DOCUMENT CODES

A. Type of Document

1.0 Research Report

- .1 Classroom
- .2 Clinical Interviews
- .3 Case Studies
- .4 Survey of Teachers
- .5 Survey of Students

2.0 Review of Research

- .1 Classroom
- .2 Clinical Interviews
- .3 Case Studies
- .4 Survey of Teachers
- .5 Survey of Students

3.0 Description of Instructional Procedure

4.0 Curriculum Content

5.0 Position Paper

- .1 Definition
- .2 Rationale and Theory
- .3 Needed Research
- .4 State of the Art

6.0 Instructional Materials

- .1 For Teachers
- .2 For students-normal class situation
- .3 For students-special learning situations

7.0 Bibliography

B. Content of Document

1.0 Place of Diagnosis in Instruction

2.0 Models of Classroom Diagnostic-Prescriptive Teaching

3.0 Instruments and Techniques for Diagnosis

- .1 Interviews
- .2 Organizational Techniques
- .3 Inventories
- .4 Analysis of Classroom Work
- .5 Affective Measures

4.0 Errors

- .1 Common
- .2 Systematic
- .3 Non-systematic

5.0 Prescriptions -- Recommendations and Rationals

6.0 Factors Contributing to Learning Difficulties

7.0 Elements of Diagnostic-Prescriptive Teaching

8.0 Teacher Education in Diagnostic-Prescriptive Teaching

APPENDIX D

ANNOTATIONS

Appendix D - Annotations

Ashlock, Robert B. Error Patterns in Computation: A Semi-Programmed Approach. Columbus, Ohio: Charles E. Merrill Publishing Co., 1972.

Designed for teachers to help children learn to improve their computation skills. Using examples taken from children's classroom work, the author provides a step-by-step method for learning to identify and analyze error patterns; he then describes instructional activities that have helped many children overcome each of these difficulties. For each set of examples of faulty computation, the reader can: (a) try to find the error pattern, (b) check to see if the diagnosis was correct, and (c) receive feedback on suggestions for corrective instruction. The author stresses the inherent dependence of computational skill on the understanding of basic mathematical concepts.

Code: A. 3.0, 6.1

B. 3.4, 4.2, 5.0

Ashlock, Robert B. Error Patterns in Computation: A Semi-Programmed Approach. Columbus, Ohio: Charles E. Merrill Publishing Co., 1976.

Essentially the same as the 1972 edition (above). The 1976 edition has a fuller discussion of "Diagnosing and Correcting Errors in Computation" (chapter 1), more examples of error patterns, and more appendices.

Code: A. 3.0, 6.1

B. 3.4, 4.2, 5.0

Ashlock, Robert B. "Structures for Teacher Education Related to Diagnosis and Remediation: Needed Research." From Proceedings of the Third National Conference on Remedial Mathematics, pp. 25-29. Kent, Ohio: Kent State University, 1976.

An examination is made of the University of Maryland's program for training teachers in the diagnosis and remediation of disabilities in mathematics. The program, built around a math remediation clinic for children, has become valuable to graduate students, for it provides opportunities for working directly with children and parents, guidance from the participating professors, and access to instructional materials. Using this program as illustrative of problems concerning teacher education, two issues are raised: (1) the need for research on teacher education models and how they affect children, and (2) the need to know in what sequence these models should be introduced to teachers working with children. In both of these areas, there is no real evidence on which to base a conclusion. The author urges professionals to share experiences and research findings.

Code: A. 3.0

B. 8.0

Ashlock, Robert B. "Underachievers in Elementary Mathematics: Diagnosing and Correcting Difficulties in Computation." Paper presented at the 51st annual meeting of the National Council of Teachers of Mathematics, Houston, Texas, April, 1973.

In working with children who have computation difficulties, the teacher has two tasks: diagnosis and corrective instruction. The most important diagnostic tool is the private interview wherein the teacher does not interfere with the child's procedure or comment on it in any way, but simply gathers data. Interviews should be conducted frequently in an attempt to offer help before error patterns are reinforced. It is important to provide a variety of response modes; knowing if a child can complete a task in the concrete, pictorial or symbolic form will help the diagnosis. Some suggestions are offered for corrective instruction in the basic facts of arithmetic and in the step-by-step procedures of arithmetic algorithms. Teachers are urged to use a variety of techniques; sometimes a child will respond to an approach simply because it is new.

Code: A. 3.0

B. 3.1

Bierden, James E. "Behavioral Objectives and Flexible Grouping in Seventh Grade Mathematics," Journal for Research in Mathematics Education I (November, 1970), pp. 207-217.

The author combined the use of intra-class grouping and behavioral objectives in designing a management plan for a mathematics class. The sample for the experiment consisted of 44 7th grade students who participated in the program at the University of Michigan laboratory school during the 1967-68 school year. The course was divided into topics, and the classroom management organized around them; students could move from one group to another as needs and interests changed. During the year computational skills of the students increased significantly. The most dramatic change, however, was the positive gain in attitudes toward mathematics. The author feels that the results of the study indicate that flexible intra-class grouping and behavioral objectives have potential for improving management procedures and increasing individualized instruction in mathematics classes.

Code: A. 1,1

B. 2.0, 3.2

Brewer, Emery. "A Survey of Arithmetic Intra-Class Grouping Practices," The Arithmetic Teacher, XIII (April, 1966), pp. 310-314.

A study was undertaken to ascertain the extent of intra-class grouping in elementary school mathematics classes, and what methods are used for forming and working with such groups. A questionnaire was sent to 2,063 elementary school teachers in 156 schools in Ohio, related research was examined, and the written opinions of professional educators evaluated. Major conclusions were that while there is sufficient use of intra-class grouping (one of three teachers) to warrant further study of its efficacy, many teachers avoid grouping because they do not understand the need for it, do not have adequate teaching materials for it, or believe that it takes too much time. The author feels that the major purpose of using intra-class grouping is to increase individualized instruction. He recommends encouraging teachers to use grouping, but adds that they must also be helped to understand its value and to learn the means for its implementation.

Code: A. 1.4

B. 3.2

Brueckner, Leo J. "Diagnosis in Arithmetic." In Educational Diagnosis, the Thirty-fourth Yearbook of the National Society for the Study of Education, pp. 269-302. Bloomington, Ill.: Public School Publishing Company, 1935.

The author discusses mathematics in a broad context, stating its four functions to be computational, informational, sociological and psychological; methods are proposed for diagnosing abilities as well as weaknesses in each of these areas. Factors contributing to growth in arithmetic ability and common symptoms of faulty learning are identified. It is stated that survey tests enable the teacher to determine the general phases of instruction that have been adequately stressed and those that need to be more fully developed. They must be followed by analytical tests for more precise diagnosis of difficulties. Any test results, however, have limited value. In order to obtain adequate data concerning the pupils' methods of work or the nature of the errors made, four general methods are prescribed: (a) observation of pupils at work, (b) analysis of written work, (c) analysis of oral responses, and (d) interviews. Principles that should underlie the organization of the curriculum in arithmetic from the point of view of both developmental and remedial teaching are presented. The author concludes with specific techniques to improve instruction for all aspects of arithmetic, with emphasis on problem-solving.

Code: A. 3.0

B. 3.0

Brueckner, Leo J. "Introduction." In Educational Diagnosis, the Thirty-fourth Yearbook of the National Society for the Study of Education, pp. 1-14. Bloomington, Ill.: Public School Publishing Company, 1935.

Educational diagnosis includes techniques for the evaluation of students' strengths, as well as weaknesses. For these purposes, there are many standard tests available, but by themselves they are not adequate diagnostic tools. Through other techniques such as observation, analysis of written work, analysis of oral responses, precise laboratory procedures, and interviews, numerous lists of the types of errors made by children have been compiled. This information provides further insight into learning difficulties, but we are still seriously handicapped by our lack of precise methods of testing and by our ignorance of the causes of most learning difficulties. We have been too concerned with the manifestations of difficulties through external symptoms. We must conduct research into their causes; to that end we need to make use of scientific technique.

Code: A. 5.0

B. 3.0, 5.4

Brueckner, Leo J. "Pedagogical Factors Associated with Learning Difficulty." In Educational Diagnosis, the Thirty-fourth Yearbook of the National Society for the Study of Education, pp. 49-62. Bloomington, Ill.: Public School Publishing Company, 1935.

Inadequacies of the instructional situation that can contribute to learning difficulties are outlined as follows: (1) inadequacy and inefficiency of instructional materials, (2) faulty or unskillful instructional practices and procedures, (3) failure of instruction to provide for individual differences, (4) ineffective guidance by teachers during learning activities, (5) undesirable personal and social relationships between teacher and pupils, and (6) ignorance of the factors contributing to learning difficulties and failure to correct them. Much research data are used to support the author's assessments of each of these problem areas which is analyzed in terms of its effects on student performance.

Code: A. 5.2

B. 6.0

Brueckner, Leo J. "The Principles of Developmental and Remedial Instruction." In Educational Diagnosis, the Thirty-fourth Yearbook of the National Society for the Study of Education, pp. 189-198. Bloomington, Ill.: Public School Publishing Company, 1935.

The consideration of numerous case studies gave rise to the author's formulation of fifteen principles that he considers basic to developmental and remedial instruction. Greatest emphasis is given to the first of these: the need to make growth of the individual the primary concern. Other principles include the observations that it is the child we are evaluating, not the subject matter; the total situation needs to be canvassed in order to determine the factors contributing to the learning difficulty, with special consideration given the emotional and social aspects; the remedial program should be viewed as an integral part of the normal classroom activities.

Code: A. 5.2, 5.3

B. 7.0

Brueckner, Leo J. "Techniques of Diagnosis." In Educational Diagnosis, the Thirty-fourth Yearbook of the National Society for the Study of Education, pp. 131-153. Bloomington, Ill.: Public School Publishing Company, 1935.

The author focuses much attention on the strengths and weaknesses of tests for diagnosing what he calls the essentials -- reading, spelling, writing and numbers. In addition to the administration of diagnostic tests, other evaluative devices must be employed, including observation of students at work, systematic analysis of students' written and oral responses, and interviewing. Lists and analyses of the kinds of errors students make in algebra and arithmetic are available. In spite of some weaknesses, this research has thrown considerable light on the nature of learning. Many remedial measures have also been identified, but little exact information concerning their effectiveness is available. More scientific technique needs to be employed in educational research.

Code: A. 3.0, 5.2

B. 3.0

Burns, Paul C., and Arnold R. Davis. "Early Research Contributions to Elementary School Mathematics," The Arithmetic Teacher, XVII (January, 1970), pp. 61-65.

Summarizes 34 pioneer researches on elementary school mathematics, published between 1919 and 1947, that touch on these topics: (a) beginning instruction; (b) content selection, (c) role of drill and practice, (d) basic operations, (e) problem solving, (f) readability and vocabulary, (g) disability, and (h) history and summaries of research. Research on disability is highlighted by the 1925 study of G. T. Buswell and Lenore John who attempted to catalog pupils' operational habits in each of the operations, and Peter L. Spencer's study of 1929-30, one of the early efforts to identify typical student errors in mathematics. All of the research cited was influenced by research in related areas; all had impact because they were closely related to the problems of the day.

Code: A. 2.0

B. 4.0, 6.0

Callahan, Leroy G. "Neuroscience and Remedial Mathematics." From Proceedings of the Third National Conference on Remedial Mathematics, pp. 38-44. Kent, Ohio: Kent State University, 1976.

An overview of the theories of brain function is given in an attempt to relate the implications of contemporary developments of neuroscience to research in remedial mathematics. Evidence suggests that there is a neurological basis for lack of performance in mathematics for some students. The number is probably relatively small, yet the need for research is evident; little investigation has been made into the mathematical behavior syndromes of such children or the necessary instructional procedures. While our top priority in the study of mathematics instruction should be on the development of excellent classroom teachers and supervisors, the clinical center must continue to be an important component in the overall program.

Code: A. 2.0, 5.2

B. 6.0

Capps, Lelon R. "Thoughts on Coordinating a Research Effort in Remediation in Mathematics." From Proceedings of the Third National Conference on Remedial Mathematics, pp. 45-47. Kent, Ohio: Kent State University, 1976.

A survey of the literature makes it apparent that in several important areas of mathematics remediation the research is very limited. There are few studies on content objectives and the learning problems associated with them. For example, several studies were found dealing with the difficulty level of basic combinations, but little or no research on causes or successful remediation techniques. Research seems to be lacking on computation with any but whole numbers, and on remediation beyond the elementary school level. It is suggested that research efforts be coordinated, either through the formation of a group or through existing channels, such as the Arithmetic Teacher. The main purpose would be to review and summarize research studies, and to disseminate information.

A skills inventory of individuals and institutions would help assess current efforts underway, and identify sources where needed research might be initiated.
Code: A. 2.0, 5.4

B. 5.0, 6.0

Carpenter, Thomas P., et al. "Notes from National Assessment: Perimeter and Area," The Arithmetic Teacher, XXII (November, 1975), pp. 586-590.

Based on studies by the National Assessment of Educational Progress and state assessments of Florida, Wisconsin and Michigan, it is concluded that by the time they reach junior high school, most students are capable of only simple measurement problems which they perform in a rote, superficial manner with little understanding of the procedures to be used. In order to increase their ability to deal with measurement, it is recommended that students should: (a) be taught to draw a picture of the figure that is under consideration, to help them visualize what is to be done; (b) be given the opportunity to verbalize plans for solving the problem; and (c) be encouraged to check answers to see if they make sense. The purpose of the above steps is to encourage students to think about measurement as a problem-solving situation rather than mechanical drill. If, in addition, they have a variety of such problems, students will be given a more meaningful treatment of measurement situations. They are also provided valuable experience with general problem-solving strategies.
Code: A. 2.5, 3.0

B. 5.0

Carpenter, Thomas P., et al. "Subtraction : What Do Students Know?" The Arithmetic Teacher, XXII (December, 1975), pp. 653-657.

The authors report national performance levels of various age groups on selected subtraction problems used in the 1972-73 mathematics assessment of the National Assessment of Educational Progress. NAEP results indicate that most students master basic subtraction with regrouping between the ages of 9 and 13. Subtraction computation improves from ages 13 to 17, even though little systematic drill is provided in the high school curriculum. Two types of errors were found most frequently: subtraction with some regrouping errors, and reversals--both of which were identified even 50 years ago as two of the most frequent subtraction errors. There is insufficient data to accurately assess whether or not students' ability to subtract is declining. Data do indicate that subtraction performance could be improved. The authors believe that improvement can be brought about by modest efforts of individual teachers if they (a) stress systematic checking of results, (b) organize instruction to develop an understanding of subtraction concepts, and (c) design an instructional program that is responsive to students' needs.
Code: A. 2.5, 3.0

B. 4.1, 5.0

Collins, Kenneth M. "An Investigation of the Variables of Bloom's Mastery Learning Model for Teaching Mathematics." Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, Ill., April, 1972.

A study was conducted with four classes to evaluate the importance of three variables of Bloom's mastery learning model: specification of objectives, use of diagnostic-progress tests, and use of alternate resources. First semester eighth grade students received four treatments that differed somewhat from those given to second semester seventh grade students. The results indicate: (1) the use of either a list of specific objectives or diagnostic prescriptive tests with recommendations is sufficient for a significant increase in student mastery of teaching objectives; (2) the use of alternate resources did not appreciably increase student achievement, an indication that the prescriptions based on the text and class-work were adequate; (3) the usefulness of specific objectives is confirmed; and (4) that general objectives of the form used have little effect on student achievement.
Code: A. 1.1

B. 2.0, 3.0, 7.0

Cook, Walter W. "Functions of Measurement in the Diagnosis and Treatment of Learning Difficulties." From "The Functions of Measurement in the Facilitation of Learning." In Educational Measurement, edited by E.F. Lindquist, pp. 33-38. Washington, D.C.: American Council on Education, 1966.

Schools organize learning experiences sequentially; therefore, in order to diagnose difficulties, it is necessary to measure a student's level of accomplishment within a given sequence. A general achievement test is designed to express this measurement in terms of a single score. It is too general to guide instruction for individual pupils. Diagnostic testing, on the other hand, is designed to reveal specific deficiencies. It generally takes two approaches: readiness testing which frequently emphasizes discovering which pupils should be placed in a given sequence, and diagnostic testing which is administered after a period of instruction. Many elementary school teachers carry on almost constant diagnosis through their teaching methods. Expertly prepared tests can make that task more thorough and efficient.
Code: A. 5.1, 5.2

B. 3.3

Cox, Linda S. Analysis, Classification, and Frequency of Systematic Error Computational Patterns in the Addition, Subtraction, Multiplication, and Division-Vertical Algorithms for Grades 2-6 and Special Education Classes. Final report, U.S. Department of Health, Education & Welfare, National Institute of Education Contract #SE018001. Kansas City: University of Kansas Medical Center, June, 1974.

Frequencies and descriptions of systematic errors in the four usual algorithms for addition, subtraction, multiplication and division of whole numbers were studied in upper-middle income, regular, and special education classrooms involving 744 children. Errors were studied within levels of computational skill for each algorithm. Results showed that five to six percent of the children made systematic errors in the addition, multiplication, and division algorithms, while 13 percent made systematic errors for the subtraction algorithm. One year later, a follow-up study indicated that 23 percent of the

same children were making either the identical or a different systematic error. The author concludes that one can say only with a qualified yes that systematic errors tend to be persistent.
Code: A. 1.5

B. 4.2

Cox, Linda S. "Diagnosing and Remediating Systematic Errors in Addition and Subtraction Computations." The Arithmetic Teacher, XXII (February, 1975), pp. 151-157.

Teachers must look for patterns in children's computational errors. Guidelines are given for constructing and administering informal diagnostic tests for systematic errors and weaknesses in the mastery of basic arithmetic facts -- two separate areas of difficulty that teachers often confuse. Basic facts, systematic errors, random errors, and careless errors are defined. While it has been demonstrated that remediation for systematic errors is possible, research on appropriate remedial teaching methods is almost nonexistent; at the present time, teachers must continue to use their own judgment.
Code: A. 3.1

B. 3.0, 4.2, 4.3

Crenson, John. "Learning Disabilities in Mathematics." From Proceedings of the Third National Conference on Remedial Mathematics, pp. 48-50. Kent, Ohio: Kent State University, 1976.

Only in recent years have mathematics educators given widespread attention to diagnosis and remediation; for the most part, research in these areas has been perfunctory and dissemination almost nonexistent. Others in the field of education have placed emphasis on the loose term, "learning disabilities," with resultant classroom procedures that are based on questionable research, as well as lacking specific application to math difficulties. Models must be created that provide strategies and techniques for diagnostic/prescriptive teaching of mathematics. This calls for a united front of mathematics educators to bring about extensive and responsible research, along with the requisite dissemination that will enable the findings to have an impact on classroom teaching.
Code: A. 5.4, 5.5

B. 3.0, 5.0

Denmark, Tom. "Diagnosis of Entry Concepts and Skills: Grades One and Two." From Proceedings of the Third National Conference on Remedial Mathematics, pp. 51-71. Kent, Ohio: Kent State University, 1976.

Prevention is viewed as the primary purpose of diagnostic teaching, particularly in the early grades. The development of a successful diagnostic program depends upon the ability to understand children's thought processes in all of their work with mathematics. Examples of first and second graders' responses to diagnostic questions are taken from 1974 and 1975 assessments made by Project for the Mathematical Development of Children. Skills that were tested include knowledge of one-to-one correspondence, counting, and

conception of equivalent sets; the latter indicated that most children had a clear conception of equivalent sets, but their definition is not the one which is supposedly taught in the school curriculum. It is emphasized that testing must make use of a variety of tasks, in order to compensate for difficulties that may arise from non-mathematical conditions such as unfamiliar vocabulary or manner of giving directions.
Code: A. 1.2

B. 4.1

Dutton, Wilbur H. Evaluating Pupils' Understanding of Arithmetic. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1964.

Although the teaching of mathematics has undergone a great deal of change, there seem to be few tools for evaluating the effectiveness of math. curricula. The author emphasizes the need for evaluation and provides evaluation instruments for both old and new math programs. Included is a review of the reasons for the development of new math programs and a review of research on evaluation procedures in mathematics. There are suggestions for the construction and use of tests to measure pupil understanding of selected arithmetical concepts in grades 3 through 6, and suggestions for informal evaluation which should be used in the day-by-day teaching of arithmetic.
Code: A. 2.0, 3.0

B. 3.1, 3.3, 3.4

Engelhardt, Jon M. "Diagnosis and Remediation in School Mathematics: Developing Continuity Among R and D Efforts." From Proceedings of the Third National Conference on Remedial Mathematics, pp. 72-77. Kent, Ohio: Kent State University, 1976.

Further investigation of mathematics diagnosis is needed regarding: (1) individual difference variables which potentially affect student performance in diagnostic situations; (2) internal structure of diagnostic testing -- type of items, response format, etc.; (3) the role of language in diagnosis; (4) teacher behavior in the diagnostic situation; and (5) examination of student errors as a diagnostic technique. Further investigation is also needed concerning the effectiveness of existing remedial programs and techniques for educating mathematics teachers. To improve continuity among all of these efforts, the following steps are suggested: the establishment of a diagnostic/remedial newsletter, the development of common literature surveys, and the establishment of common data collection across mathematics clinics.
Code: A. 5.3

B. 3.0, 5.0

Glennon, Vincent J., and John W. Wilson. "Diagnostic-Prescriptive Teaching." In The Slow Learner in Mathematics, the Thirty-fifth Yearbook of the National Council of Teachers of Mathematics, edited by W. C. Lowry, et al., pp. 282-318. Washington, DC: National Council of Teachers of Mathematics, 1972.

In this discussion of diagnostic-prescriptive teaching of mathematics for

slow learners, the problem is approached in a broad context, considered to be "as large as a theory of instruction itself," and takes into account the numerous causes of learning difficulties in addition to those that are cognitive. A taxonomy for the content of elementary school mathematics is presented, with the work of learning theorists Gagne and Bloom discussed. Group techniques for diagnostic-prescriptive teaching have emphasized product, while individual methods emphasize process; both approaches are necessary, but we have overemphasized the former. Lessons are presented from a case study in which the content taxonomy was used by the teacher as a guide in the selection of objectives and of relevant group and individualized teaching procedures.

Code: A. 2.0, 5.1, 5.2

B. 1.0, 2.0, 5.0

Hynes, Michael C. "Response Modes and Diagnostic Procedures: Needed Research." From Proceedings of the Third National Conference on Remedial Mathematics, pp. 78-82. Kent, Ohio: Kent State University, 1976.

Researchers agree that children move sequentially from concrete response modes in mathematical performance to symbolic modes, with intervening stages. Techniques for diagnosing the various response modes do exist, but we have little knowledge for determining effective sequence for testing. A product-oriented philosophy uses other response modes only for those skills judged to be unsatisfactory by initial testing done at the abstract level. A process-oriented philosophy uses all response modes for testing, regardless of student strengths or weaknesses, but might use either sequence -- abstract to concrete, or the reverse. Four levels of diagnosis used at the Arithmetic Center at the University of Maryland suggest an overall abstract to concrete testing sequence, but it is not always clear whether specific diagnostic activities should begin on the abstract or the concrete level. Research is needed if we are to understand the most appropriate sequence for producing information which allows the effective prescription of remedial instruction.

Code: A. 5.2, 5.3

B. 5.0

Irons, Jerry L. "The Need for Packaged Material and a Delivery System." From Proceedings of the Third National Conference on Remedial Mathematics, pp. 84-91. Kent, Ohio,: Kent State University, 1976.

Many resource teachers and diagnosticians are sorely lacking in the knowledge of specific diagnostic instruments and techniques pertaining to mathematics. Therefore, diagnostic tools must be developed and disseminated. Critical components of a diagnostic package must be the scope of the mathematics to be tested, sequencing of the mathematical concepts, levels to be tested, a resource list of recommended alternate instruments and activities, and the structured interview and/or information sheet. Development of a delivery system must include a system for training teachers and a means of studying the effectiveness of various systems. The Stephen F. Austin State University in Texas offers a model for teacher training. Key elements of the success of the program were the supportive roles of the school superintendents and the high standards maintained for the trainees. While the requirements of a

diagnostic package and delivery system exceed our present abilities, it is argued that the academic needs of many children call for our efforts to move in this direction.
Code: A. 5.0

B. 3.0

Johnson, Martin L. "The Role of Piagetian Theory and 'Piagetian-Type' Tasks in Mathematics Diagnosis and Remediation." From Proceedings of the Third National Conference on Remedial Mathematics, pp. 30-37. Kent, Ohio: Kent State University, 1976.

The usefulness of Piagetian theory to diagnostic and remedial instruction of mathematics is examined, particularly at the Concrete Operational level of children's cognitive development. Three examples are provided of the tasks that Piaget developed to help evaluate children's degree of understanding for certain arithmetical and geometrical ideas. It is concluded that knowledge of Piagetian theory would be helpful to a clinician in designing instructional activities; for example, knowing that concrete operational children reason on objects and events that are real to them should assist in choosing appropriate learning experiences. More research is needed, but Piagetian theory has already demonstrated usefulness, one of its major implications to current mathematics instruction being the concept of gathering information from interviews.
Code: A. 5.0

B. 3.0, 5.0, 6.0

Lankford, Francis G., Jr. Some Computational Strategies of Seventh Grade Pupils. Final Report, U.S. Department of Health, Education, and Welfare, Office of Education National Center for Educational Research and Development (Regional Research Program) and The Center for Advanced Study, The University of Virginia, Project Number 2-C-013, Grant Number OEG-3-0035. Charlottesville, Virginia: University of Virginia, October, 1972.

Diagnostic interviews were conducted with 176 seventh-grade students from six schools in Virginia, Georgia, Colorado, Michigan, and Washington, DC. Students "thought aloud" while computing 13 whole number and 15 fraction exercises and 8 comparison questions (e.g., Which is larger, 2/3 or 5x1x1?). Among other conclusions is that students vary widely in their computational strategies. These strategies are discussed, along with the nature of the wrong answers that were given, and some of the characteristics of good and poor computers. Descriptions of the tape-recorded interviews are included in the report. Recorded interviews are seen as a promising technique for identifying computational strategies. It is urged that children be encouraged to reveal their individual strategies and that teachers recognize these as original thinking rather than deviations from a desired norm.
Code: A. 1.2

B. 4.1, 4.2

Lankford, Francis G., Jr. "What can a Teacher Learn About a Pupil's Thinking Through Oral Interviews?" The Arithmetic Teacher, XXI (January, 1974), pp. 26-32.

A summary of the author's earlier study, Some Computational Strategies of Seventh Grade Pupils, Final Report, U.S. Department of Health, Education, and Welfare, 1972. See file #5.

Code: A. 1.2, 3.0

B. 4.1, 3.1

McAloon, Ann. "Some Issues of Concern in the Development of a Diagnostic Testing Program." From Proceedings of the Third National Conference on Remedial Mathematics, pp. 92-98. Kent, Ohio: Kent State University, 1976.

An outline is provided of important considerations for the development of a diagnostic testing program appropriate for use by classroom teachers. Questions must first be raised concerning the content -- what objectives or skills are considered necessary. Development of a diagnostic test requires basically two steps: analysis of the complex performance into its component subskills, and making the test as free as possible from any other source of difficulty. Factors other than mathematical ability must be taken into account, i.e., emotional needs, maturity of the student, cultural background, etc. General criteria of tests are listed. It is assumed that as a group, elementary school teachers are inadequately trained in mathematics and educational measurement. We need to ask in what areas they need the most help, when that help can be given and by whom.

Code: A. 3.0, 5.0

B. 3.3, 8.0

O'Brien, Thomas C., and June V. Richard. "Interviews to Assess Number Knowledge," The Arithmetic Teacher, XVIII (May, 1971), pp. 322-326.

Reference is made to the research of Jean Piaget in which it is maintained that knowledge is a process rather than a product. The authors suggest that if such is the case, then traditional paper-and-pencil methods of assessing student abilities are less valuable for teachers than interviews which place more emphasis on thought processes than on factual information. The authors report on what they call "the first stage of an interview protocol" which was used to evaluate counting, simple addition and missing addend knowledge of first-grade pupils. Through excerpts from actual interviews, children's strategies in solving five different tasks are described. The technique is offered only as a beginning step in designing a good diagnostic interview strategy. The interviews were conducted as part of a larger research project; no data are reported here.

Code: A. 1.2

B. 3.1

Peck, Donald, M., and Stanley M. Jencks. "What the Tests Don't Tell." The Arithmetic Teacher, XXI (January, 1974), pp. 54-56.

Interviews with students often permit the diagnosis of conceptual difficulties in mathematics that are not revealed through paper and pencil tests. An

example is given of sixth-grade students who had solved a fraction problem correctly but were unable to explain what they had done or what it meant. Later, through a series of teacher-posed questions, one of the students was guided toward the reasoning that enabled her to solve the same problem with understanding; she even drew a picture to defend her conclusion. Teachers can gain insight into children's depth of understanding by having them talk about the process while solving a problem, and demonstrating knowledge through the organization of familiar objects.

Code: A. 3.0

B. 3.1

Pincus, Morris, et al. "If You Don't Know How Children Think, How Can You Help Them?" The Arithmetic Teacher, XXII (November, 1975), pp. 580-585.

The assumption that a child's mistakes in computation are caused by insufficient knowledge of the basic arithmetic facts is often erroneous. Careful analysis of errors through observation and interview is essential in order to accurately diagnose the difficulties. An elementary school mathematics committee looked for common errors for each of the basic operations, and devised a series of diagnostic exercises for children in grades 4, 5, and 6. They found that questioning children individually was the most effective diagnostic device. Difficulties that were found common to the four basic operations, as well as those that were specific to addition, subtraction, multiplication or division, are listed. Every item is followed by a number of recommendations for remediation. The committee's report emphasizes the need for understanding each child's thought processes if an effective program of individualized instruction is to take place.

Code: A. 1.1, 3.0

B. 3.1, 4.1, 5.0

Reisman, Fredericka K. A Guide to the Diagnostic Teaching of Arithmetic. Columbus, Ohio: Charles E. Merrill Publishing Company, 1972.

This is a manual for preservice and in-service teachers dealing with techniques for teaching mathematics from kindergarten to the upper elementary grades. Examples are given of children's difficulties with addition, subtraction, multiplication, division, and work with fractions. There is discussion of the skills involved in doing arithmetic, and of ways to evaluate a student's strengths and weaknesses, including guidelines for using standardized mathematics inventories and for preparing one's own informal inventories. Teachers are provided with sample lesson plans, analysis of common errors children make in elementary school mathematics, suggestions for remediation, and mini-case studies for practicing diagnostic skills. Practical applications of prominent educators' and psychologists' hierarchies are presented, including those of Bruner, Piaget, Gagne, Brownell, Krathwohl, Bloom, Rogers and Maslow.

Code: A. 6.1

B. 3.0, 4.0, 5.0, 6.0

Riedesel, C. Alan. "A Diagnostic Instrument for Mathematics; The Affective Domain; A First Step." From Proceedings of the Third National Conference on Remedial Mathematics, pp. 99-109. Kent, Ohio: Kent State University, 1976.

The author speculates that the lowering of math scores for both elementary and high school students may be at least partly related to the lack of social applications, games, and a "fun" approach during the new mathematics era, which may have contributed significantly to a decrease in children's interest. A diagnostic instrument was designed to assess the reasons for children's enjoyment or lack of enjoyment in working with mathematics. The 99-item questionnaire is based on the belief that if we can find the reasons for a child's disinterest in mathematics, we may be able to contribute to improved achievement. No data are presented; the questionnaire is offered for use by others with the hope that their responses will both refine and evaluate the instrument as a diagnostic tool.

Code: A. 6.2

B. 3.5

Roberts, Gerhard H. "The Failure Strategies of Third Grade Arithmetic Pupils," The Arithmetic Teacher, XV (May, 1968), pp. 442-446.

A study of 766 third graders was conducted in order to determine the computational skills in which pupils were most deficient and to analyze their strategies that led to failure. Four major error categories were noted: use of the wrong operation, error in recalling basic number facts, improper use of the algorithm, and responses that showed no discernible relationship to the given problem. Percentages of errors caused by each of these failure strategies are given. The largest number of errors involved incorrect application of the algorithm. Purely careless numerical errors were fairly consistent throughout the four ability levels of the children studied, but faulty algorithm occurred far more frequently in the low ability group. Each failure strategy is discussed, and steps for remediation are suggested. The authors conclude by saying that teachers must analyze children's computation methods in order to provide the necessary help.

Code: A. 1.6, 3.0

B. 4.2, 4.3, 5.0

Romberg, Thomas A. "The Application of 'Process Evaluation' to Corrective Instruction: A Suggestion." From Proceedings of the Third National Conference on Remedial Mathematics, pp. 110-121. Kent, Ohio: Kent State University, 1976.

The conventional deductive method of inquiry is not appropriate to the study of most corrective instructional events. It is suggested that "process evaluation" is a better methodology to be applied to corrective mathematics instruction; it is a descriptive and inductive search aimed at raising questions about an event rather than drawing conclusions. In "process evaluation," effects are broadly conceived, for it is not realistic

to focus on only one desired outcome; the time frame for data gathering includes pertinent information from before, during, and after the instructional event; descriptions of corrective instruction are constructed from the collected data; and interpretations are formulated from the descriptions to identify major relationships. "Process evaluation" expands the notion of effects beyond direct intended effects to a search for others that are unintended.

Code: A. 5.0

B. 3.0, 5.0

Skarbek, James F. "Diagnostic Analysis of Mathematics Skills." In The Slow Learner in Mathematics, the Thirty-fifth Yearbook of the National Council of Teachers of Mathematics, edited by W. C. Lowry, et al., pp. 513-516. Washington, DC: National Council of Teachers of Mathematics, 1972.

In order to illustrate oral analysis as a diagnostic technique in determining mathematical skills, the explanations of four fourth-grade students, as each performed a subtraction task, are given. The computations are reproduced as they appeared on the children's papers. In each case, the author diagnoses the student's work and indicates when further questioning is needed.

Code: A. 3.0

B. 3.1, 3.4

Smith, Robert F. "Diagnosis of Pupil Performance on Place-Value Tasks," The Arithmetic Teacher, XX (May, 1973), pp. 403-408.

A study was undertaken to identify skills prerequisite to the mastery of place-value tasks involving hundreds, tens, and ones, and to determine which of these skills had not been mastered by primary grade children. After 12 prerequisite skills were identified, five place-value diagnostic tests were designed and administered to 323 second-grade students in four parochial schools in Brooklyn. The skills measured on each test are listed, and sample items included. Skills that gave students difficulty are itemized. Renaming numerals proved to be troublesome for both high and low achievers, a finding that has also been reported by other research. It is suggested that in order to ensure pupil mastery of basic skills prerequisite to successful performance on more complex place-value tasks, teachers must know what those skills are, teach them in proper sequence, diagnose each student's mastery of them, and adjust instruction accordingly.

Code: A. 2.1

B. 3.3, 4.1, 7.0

Stake, Robert E., and Terry Denny. "Needed Concepts and Techniques for Utilizing More Fully the Potential of Evaluation." In Educational Evaluation: New Roles, New Means, the Sixty-eighth Yearbook of the National Society for the Study of Education, edited by R. W. Tyler, pp. 370-390. Chicago, Ill.: The University of Chicago Press, 1969.

Current diagnostic tools are not adequate for prescribing appropriate treatment.

Our concept of diagnosis must be expanded to include prescription as well as assessment. Educational evaluation should include programs as well as student performance. Specialists are increasingly needed for a variety of evaluative tasks, including helping persons and institutions convey their purpose; assessing instructional materials and classroom instruction; describing student performance; and guiding the selection of diagnostic tools appropriate to given tasks. Suggestions for strengthening evaluation methods are discussed, and a variety of strategies and research findings are included.
Code: A. 5.0

B. 3.0

Stenquist, John L. "The Administration of a Program of Diagnosis and Remedial Instruction." In Educational Diagnosis, the Thirty-fourth Yearbook of the National Society for the Study of Education, pp. 269-302. Bloomington, Ill.: Public School Publishing Company, 1935.

Emphasis is placed on the need for education to be responsive to the wide variation among children, in order to insure that all pupils will receive equal opportunity. An effective program of diagnosis, under capable leadership, is essential to this end. It should be characterized by: (1) good organization of personnel, guided by a common philosophy; (2) a research program, which includes teachers and administrators, to keep up with new developments and to guide the diagnostic work of the schools; (3) administrative measures that facilitate diagnostic teaching; (4) teacher-training; (5) selection of good instructional materials; and (6) careful record-keeping.
Code: A. 5.0

B. 7.0

Thelen, Herbert A. "The Evaluation of Group Instruction." In Educational Evaluation: New Roles, New Means, the Sixty-eighth Yearbook of the National Society for the Study of Education, edited by R. W. Tyler, pp. 115-155. Chicago, Ill.: The University of Chicago Press, 1969.

Diagnosis is part of all instruction, but its purposes and strategies differ, depending on the nature and expectations of the classroom group. Three basic views of classrooms are described: (1) collectivity, where children are taught essentially the same things at the same time and where diagnosis attempts to ascertain where individual students stand with respect to specific learning goals; (2) an interpersonal network, where children are seen in more individual terms and where diagnosis considers the private needs that may be diverting the individual's energy away from required tasks; and (3) a microsociety where diagnosis is concerned with full functioning of everyone. In the author's view, only the vision of the classroom as a microsociety can deal with the student in all of these roles -- as learner, individual, and group member. If we accept the concept of the classroom as an educative community, then diagnosis can describe situtations with which individuals can cope, rather than just measure fragmentary skills that are assumed to meet everyone's needs.
Code: A. 5.2

B. 1.0

Trueblood, Cecil R. "A Model for Using Diagnosis in Individualizing Mathematics Instruction in the Elementary School Classroom," The Arithmetic Teacher, XVIII (November, 1971), pp. 505-511.

Trueblood argues that the current emphasis on planning for individual differences has changed instruction from a "means-referenced" (How should I teach?) to a goal-referenced point of view (What should I teach?). Four types of individualization are briefly described: Individually Prescribed Instruction (IPI), Personalized Instruction, Self-directed Instruction, and Independent Study. The author points out that for each of these, teachers require continuing information about the learner. A goal-referenced diagnosis model (What specific behaviors do my learners possess?) is included for classroom use of IPI. The model outlines tasks and their sequence, but no examples are given. Trueblood suggests some ways in which the model would need to be modified for the PI, SDI, and SI context.
Code: A. 3.0

B. 2.0

Tyler, Ralph W. "Characteristics of a Satisfactory Diagnosis." In Educational Diagnosis, the Thirty-fourth Yearbook of the National Society for the Study of Education, pp. 95-111. Bloomington, Ill.: Public School Publishing Company, 1935.

Effective diagnosis must have worthwhile objectives, validity, and reliability; it must provide comparable data and sufficiently exact data; it must be comprehensive, practicable, reasonably objective, appropriate to the program, carried to a satisfactory level of specificity, and conducted by qualified people. These characteristics are viewed as essential in evaluating learning abilities in general; mathematics is not singled out for discussion except in a reference to the diagnostic tool of interviewing.
Code: A. 5.2

B. 1.0, 3.1

Weaver, J. Fred "Big Dividends from Little Interviews," The Arithmetic Teacher, II (April, 1955), pp. 40-47.

The interview, wherein children "think out loud" as they respond to specific mathematical problems, is a fruitful instructional procedure that can be used by any classroom teacher. As a method of diagnosis that is more revealing than written answers alone, it can be used to determine different levels of ability within one class and to evaluate change in an individual student's ability. The knowledge gained from interviews can be used to assist the teacher in developing a differentiated instructional program to meet the varying needs of the students. An example is provided by describing interviews with fourth grade pupils to determine their understanding of multiplication.
Code: A. 3.0

B. 3.1

West, Tommie A. "Diagnosing Pupil Errors: Looking for Patterns," The Arithmetic Teacher, XVIII (November, 1971), pp. 467-469.

Diagnosis that looks for patterns of errors rather than random errors should be the teacher's goal, for patterns reveal the conceptual difficulties children are having, their insufficient understanding of the procedures they are trying to use. Diagnostic teaching requires four abilities: (a) distinguishing between conceptual and careless errors, (b) identifying the precise nature of the careless errors, (c) inferring the cause of the conceptual errors, and (d) prescribing remedial procedures. Samples of written work of three children are also analyzed.

Code: A. 3.0

B. 4.2, 7.0